P.R.GOVERNMENT COLLEGE (A), KAKINADA

B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.e.f. 2017-18)

Course : CLUSTER VIII- (A,B)-2 SPECIAL FUNCTIONS

No. Hours: 75 hrs **Credits: 05**

Objectives:

- To understand the concepts of special functions which have applications in Physical Sciences.
- To learn finding power series solutions to some special types of differential equations.

UNIT - I : HERMITE POLYNOMIAL **(15 hrs)**

Hermite Differential Equations, Solution of Hermite Equation, Hermite's Polynomials, Generating function, Other forms for Hermite Polynomial, To find first few Hermite Polynomials, Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials. (CHAPTER: 6.1 to 6.8)

UNIT - II : LAGUERRE POLYNOMIALS **(15 hrs)**

Laguerre's Differential equation, Solution of Laguerre's equation, Laguerre Polynomials, Generating function, Other forms for the Laguerre Poynomials, To find first few Laguerre Polynomials, Orthogonal property of the Laguerre Polynomials, Recurrence formula for Laguerre Polynomials, Associated Laguerre Equation. (CHAPTER: 7.1 to 7.9)

UNIT – III : LEGENDRE'S EQUATION **(15 hrs)**

Definition, Solution of Legendre's Equation, Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation(derivationis not required), To show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{1}/_{2}$, Orthogonal properties of Legendre's Equation, Recurrence formulae, Rodrigue's formula. (CHAPTER: 2.1 to 2.8, 2.12)

UNIT - IV : BESSEL'S EQUATION **(15 hrs)**

Definition, Solution of Bessel's General Differential Equations, General solution of Bessel's Equation, Integration of Bessel's equation in series for n=0, Definition of $J_n(x)$, Recurrence formulae for $J_n(x)$, Generating function for $J_n(x)$. (CHAPTER: 5.1 to 5.7)

UNIT - V : BETA AND GAMMA FUNCTIONS **(15 hrs)**

Euler's Integrals - Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions,Another form of Beta Function, Relation between Beta and Gamma Functions, Other Transformations. (CHAPTER: 2.9 to 2.15)

Prescribed text book: Special Functions by J.N. Sharma and Dr. R.K. Gupta.

BLUE PRINT FOR QUESTION PAPER PATTERN,

SEMESTER-VI, CLUSTER VIII –A, B, D-2

SPECIAL FUNCTIONS

UNIT	TOPIC	V.S.A.Q 1 M	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	Hermit Polynomial	01	01	01	14
II	Laguerre Polynomial	01	01	01	14
III	Legendre's Equation	01	01	02	22
IV	Bessel's Equation	01	01	02	22
V	Beta And Gama Functions	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P.R. Government College (A), Kakinada

III B.Sc. Degree Examinations: Semester-VI, Mathematics

COURSE (Cluster VIII (A,B,D) 2) Special Functions

PAPER-VIII A,B, D 2 (MODEL PAPER w.e.f. 2019-2020)

Time: 2 hrs 30 Min Max. Marks : 60 M

PART – I

Answer ALL the following questions. Each question carries 1 mark. 5 x 1 = 5 M

1. Write the generating function of Hermit's polynomial.
2. Show that $L_1(x) = 1 - x$.
3. Define Legendre's equation.
4. Write $J_0(x)$.
5. Show that $\Gamma(1) = 1$.

PART – II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15M

6. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)dx$.
7. Show that $L_2(x) = \frac{1}{2!}(2 - 4x + x^2)$.
8. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.
9. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
10. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}$ dx.

PART – III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. 5 X 8 = 40 M

SECTION – A

11. State and Prove Rodrigue's formula for $H_n(x)$.
12. Prove that $xL_n''(x) + (1 - x)L_n'(x) + nL_n(x) = 0$.
13. Prove that $(2n + 1)xP_n = (n + 1)P_{n+1} + nP_{n-1}$.
14. Show that $\int_{-1}^{1} P_m(x).P_n(x)\, dx = 0 \; if \; m \neq n$.

SECTION – B

15. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
16. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
17. When n is a positive integer, prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5.....(2n-1)}$
18. Prove that $B(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$